

Universität Jena

Fakultät für Mathematik und Informatik - Institut für Mathematik
Oberseminar "Analysis, Geometrie & Dynamik"

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Numerical computation of rotation sets of torus homeomorphisms

by K. Polotzek (joint work with K. Padberg-Gehle and T. Oertel-Jäger)
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Abstract

The rotation theory of mappings $f : \mathbb{T}^m \rightarrow \mathbb{T}^m$ on the m -dimensional torus $\mathbb{T}^m := \mathbb{R}^m / \mathbb{Z}^m$ focusses on limits of sequences of the form

$$\left(\frac{F^{n_i}(x_i) - x_i}{n_i} \right)_{i \in \mathbb{N}}$$

with respect to a lift $F : \mathbb{R}^m \rightarrow \mathbb{R}^m$ of f and with $x_i \in \mathbb{R}^m, n_i \in \mathbb{N}, i \in \mathbb{N}$ and $n_i \rightarrow \infty$ as $i \rightarrow \infty$. The set of all accumulation points of such sequences is called the *rotation set* $\rho(F)$ of F . For homeomorphisms of the two-torus, homotopic to the identity, this set is known to be compact and convex [2] and, generically, it appears polygonal [3].

As its rotation set provides strong information about the system's behavior but, in general, is not given analytically, one is interested in a numerical approximation of this set. In the talk, direct approaches [1] to this task are presented in order to illustrate the limits of a pointwise detection of rotation sets. Alternatively, we formulate a set-oriented algorithm and obtain good numerical representations of the rotation sets for several example maps.

- [1] Pierre-Antoine Guiheneuf, How roundoff errors help to compute the rotation set of torus homeomorphisms, eprint arXiv:1406.2254 (06/2014)
- [2] Michał Misiurewicz and Krystyna Ziemian, Rotation sets for maps of tori, J. London Math. Soc. (2) 40 (1989), no. 3, 490–506. MR 1053617 (91f:58052)
- [3] Alejandro Passeggi, Rational polygons as rotation sets of generic homeomorphisms of the two torus, J. London Math. Soc., jdt040 (2013).