Spectral Approximation of Schrödinger Operators: Continuous Behavior of the Spectra

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The first quasicrystals where discovered by Dan Shechtman in the year 1984. Later 1987 [TIM] discovered a quasicrystalline structure in an Aluminum-Copper-Iron composition. By changing the concentration of the chemical elements it produces stable quasicrystaline structure out of periodic structures. From the mathematical point of view the study of the associated Schrödinger operators turns out to be a challenging question. Up to know the most is known for the one-dimensional case where the use of transfer matrices is a powerful tool. According to [TIM], it is natural to ask whether Schrödinger operators related to aperiodic structures can be approximated by periodic ones. The answer for this question is not clear if the operator depends on the local structure of the aperiodic system. In particular, if the Schrödinger operator arises by pattern equivariant functions of an aperiodic system. We provide a strategy to find a periodic approximation such that the spectra converges. It is remarkable that this strategy mainly uses the structure of the associated dynamical systems and is independent of the dimension.

Given a family of linear, self-adjoint, (bounded) operators $(A_t)_{t\in\mathcal{T}}$ indexed by a topological space we will address the question under which assumptions on $(A_t)_{t\in\mathcal{T}}$ the map $\Phi : t \in \mathcal{T} \mapsto \sigma(A_t)$ varies continuously with respect to the Hausdorff metric on $\mathbb{R}$. The continuity with respect to the operator norm is in general to restrictive whereas the strong operator topology is not sufficient. In [BB] the notion of (p2)-continuity is introduced. This condition characterizes the continuity of the map Φ. Furthermore, if $\mathcal{T}$ is a metric space the Hölder continuity of Φ is characterized leading to an interesting observation.

After the discussion of the general theory, we provide a tool to prove the continuous behavior of the spectra. This tool can be used if the operators are related to a dynamical system or more general to a groupoid. This is, for instance, the case for Schrödinger operators.

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[TIM] A. P. Tsai, A. Inoue, T. Masumoto, *A Stable Quasicrystal in Al-Cu-Fe System*, Jap. J. Appl. Phys. **26** (1987), L1505-L1507.